

Limits on the location of planetesimal formation in self-gravitating protostellar discs

C.J. Clarke¹ and G. Lodato²

¹*Institute of Astronomy, Madingley Rd, Cambridge, CB3 0HA, UK*

²*Dipartimento di Fisica, Università degli studi di Milano, Via Celoria 16, Milano, I-20133, Italy*

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ABSTRACT

In this Letter we show that if planetesimals form in spiral features in self-gravitating discs, as previously suggested by the idealised simulations of Rice et al, then in realistic protostellar discs, this process will be restricted to the outer regions of the disc (i.e. at radii in excess of several tens of A.U.). This restriction relates to the requirement that dust has to be concentrated in spiral features on a timescale that is less than the (roughly dynamical) lifetime of such features, and that such rapid accumulation requires spiral features whose fractional amplitude is not much less than unity. This in turn requires that the cooling timescale of the gas is relatively short, which restricts the process to the outer disc. We point out that the efficient conversion of a large fraction of the primordial dust in the disc into planetesimals could rescue this material from the well known problem of rapid inward migration at a \sim metre size scale and that in principle the collisional evolution of these objects could help to re-supply small dust to the protostellar disc. We also point out the possible implications of this scenario for the location of planetesimal belts inferred in debris discs around main sequence stars, but stress that further dynamical studies are required in order to establish whether the disc retains a memory of the initial site of planetesimal creation.

Key words: accretion, accretion discs - planetary systems: formation - hydrodynamics - instabilities

1 INTRODUCTION

A much debated aspect of protostellar disc evolution concerns the way that the solid component of the disc (for which the range of grain sizes is initially in the sub-micron range as in the interstellar medium; Dullemond et al., 2007) is assembled into larger bodies. This process is not only evidenced by the range of rocky/icy bodies (terrestrial planets, comets, asteroids) in our Solar System, but also by the existence of dusty debris around young main sequence stars, which are interpreted in terms of the grinding down of a reservoir of rocky/icy planetesimals of km size or more (Wyatt & Dent, 2002; Greaves et al., 2004; Wyatt, 2008). Clearly, therefore, large rocky/icy bodies are in place by an age of around 10 Myr (corresponding to the youngest debris discs) but this does not in itself provide any constraints about *when* these objects are assembled during the preceding, gas rich phase of disc evolution. Spectral evidence of grain growth has however been accumulating in recent years, as the form of the spectral energy distribution at mm wavelengths and beyond appears to require the existence of emitting particles of at least cm scales in some objects (e.g., Testi et al. 2003). The recent detection of emission at a wavelength of 10 cm in the protostellar disc system HL Tau is particularly striking in this regard (Greaves et al., 2008): although the extent to which this emission is contaminated by non-thermal (free-free) emission is debatable,

the detection of thermal emission at this wavelength requires substantial grain growth (to > 10 cm) in a system that is still young (< 1 Myr old) and where the disc is massive ($> 0.1M_{\odot}$). Apparently, then, grain growth is already under way in early evolutionary stages when the disc is still strongly *self*-gravitating.

Grain growth however presents a potential problem for the retention of solid material in the disc, since objects of metre size are subject to strong radial migration as a result of gas drag (Weiden-schilling, 1977). For example, in a massive axisymmetric disc, the predicted timescale on which metre sized bodies would be swept into the star from a radius of 5 AU is \sim a few 10^3 years (cf. Rice et al., 2004) and there is therefore the concern that the disc could be severely depleted in solid material during the early self-gravitating phase of disc evolution. This would be at odds with observational evidence from debris discs that at least some objects retain several tens of earth masses of rocky/icy debris after the disappearance of the disc gas, and would also severely impact the planet formation potential of the disc.

Rice et al. (2004) however suggested a way that this outcome could be circumvented, by distinguishing the rapid radial migration that occurs in a massive *axisymmetric* disc from the behaviour expected when one takes into account the strong *spiral* features in massive, self-gravitating discs. Their simulations demonstrated that

in this case the effect of gas drag is to strongly concentrate objects of around metre size in the pressure maxima associated with spiral shocks. In the simulations, this concentration resulted in a more than hundredfold increase of the local solid density, bringing the simulations into the regime, for which they were not designed, where the disc is locally dominated by the mass in solids rather than in gas. Rice et al. (2006) however estimate that the conditions in the arms are not only conducive to runaway grain growth by collisions but can also lead to the creation of large (km scale) planetesimals through the action of self-gravity in the solid phase (see also Goldreich & Ward 1973, Youdin & Shu 2002). If this is indeed the case, then it effectively ‘rescues’ the solids from rapid radial migration, since gas drag is unimportant for such large objects. This scenario then raises the possibility of storing some fraction of the solids in self-gravitating discs (which initially total several hundred earth masses of solids) in the ‘safe’ form of planetesimals. In principle, these planetesimals could then be available for involvement in future planet building, as well as providing a reservoir for dust production in future debris discs.

Such a picture is only workable, however, if a number of conditions are met, regarding both the viability of planetesimal formation in self-gravitating discs, and the subsequent evolution of the planetesimal swarm. In this Letter we concentrate on the former issue: although Rice et al. (2004) demonstrated the viability of dust concentration in spiral arms in their simulations (which employed a simple scale free ‘toy’ cooling model for the disc gas), we here explore what are the conditions required for this mechanism to work in discs with realistic cooling. Our conclusions will be used to provide initial conditions for future calculations of planetesimal evolution in self-gravitating discs.

In Section 2 we explain how the viability of solid growth in spiral features depends on the dust concentration timescale as compared with the (roughly dynamical timescale) lifetime of individual spiral features. The dust concentration timescale is linked to the amplitude of spiral features in the gas, which itself depends on the cooling timescale of the gas. In section 3 we then use analytic models of the structure of self-gravitating discs subject to realistic cooling in order to demonstrate that the concentration of dust in spiral features is feasible only in the outer regions of protoplanetary discs (beyond a few tens of AU). In Section 4 we discuss the implications of this result for planet formation models and for the retention of dust grains in protostellar discs. We emphasise that other mechanisms may also be efficient in the rapid production of planetesimals (see e.g. Ciesla 2009, Brauer et al 2008, Kretke & Lin 2007) and that our conclusions here apply only to the viability of planetesimal assembly in self-gravitating discs.

2 THE LINK BETWEEN DUST CONCENTRATION EFFICIENCY AND COOLING TIMESCALE

The success of numerical simulations in achieving the desired concentration of dust in spiral features is at first sight surprising, given the fact that in self-gravitating gas discs it is well established that individual spiral features are transient (though regenerative) features (e.g., Lodato & Rice, 2004, Britsch et al., 2007). The characteristic lifetime for spiral features is of order the dynamical timescale (Ω^{-1}) and so the fact that spiral arms are able to concentrate dust successfully suggests that the concentration timescale must also be $< \Omega^{-1}$. Rice et al. (2004) present simple arguments that this should indeed be the case; these arguments however rely on an assumption that the spiral features involve pressure varia-

tions of order unity, which is indeed the case in these particular simulations. Nevertheless, our recent simulations demonstrate that the amplitude of spiral features is itself a function of disc cooling timescale (Cossins et al., 2009); the observed dependence (whereby the fractional amplitude of spiral features scales as the inverse square root of the ratio of cooling timescale to dynamical timescale) may be simply understood in terms of the properties of weak adiabatic shocks. This means that in practice *the concentration of dust in spiral features will be restricted to regions of the disc where the cooling timescale is appropriately short.*

2.1 The link between dust concentration efficiency and spiral arm amplitude

The concentration of dust in spiral arms is effected by the same mechanism that is involved whenever dust concentrates in pressure maxima in gas discs (e.g. Haghighipour & Boss 2003, Godon & Livio 2000, Klahr & Bodenheimer 2003): this same mechanism has been invoked in cases where a variety of physical processes (e.g. vortices or edges induced by planets/binary companions) are responsible for the creation of the local pressure maximum. In each case, dust concentration relies on the fact that the gas is partially pressure supported and thus its local orbital frequency is respectively sub (super)- Keplerian outside (inside) of the pressure maximum. Dust particles behave ballistically to first order and thus orbit respectively faster (slower) than the gas outside (inside) the pressure maximum. If one now introduces drag forces between the dust and the dominant gas component, the dust is then decelerated (accelerated) by the gas outside (inside) the pressure maximum and therefore moves inwards (outwards). Thus dust accumulates preferentially at the location of pressure maxima.

The magnitude of the radial velocity induced by drag on the solid component depends on the ratio between the stopping time t_s and the dynamical time Ω^{-1} (see, for example, Takeuchi & Lin, 2002). In the limit of weak drag forces (i.e. where the stopping timescale t_s is much longer than the local orbital time Ω^{-1}) then the rate of radial migration simply increases with increasing drag strength (i.e. it scales as $1/\Omega t_s$). If however the drag is very strong ($\Omega t_s \ll 1$) then the rate of radial migration is now set by the terminal velocity of the grain’s radial flow, which now scales as Ωt_s (i.e. decreases with increasing drag strength). We thus have

$$v_{\text{rad}} \sim \frac{\Delta v}{\Omega t_s + 1/\Omega t_s}. \quad (1)$$

The maximum radial velocity is acquired at intermediate drag strength when $\Omega t_s \sim 1$, which corresponds roughly to particles of metre size in the case of the massive (self-gravitating) discs considered here (Rice et al 2004). Since in this case the grain is brought into co-rotation with the gas over each orbital period, then the radial velocity acquired is roughly the difference in orbital velocity (Δv) between a ballistic grain and the local gas velocity.

This velocity difference Δv can be readily assessed by consideration of radial force balance for the gas, where the difference between centripetal acceleration and gravitational acceleration is provided by the acceleration due to the local pressure gradient. Thus

$$v_\phi^2 = v_K^2 + \frac{R}{\rho} \frac{\partial P}{\partial R}, \quad (2)$$

where v_K is the Keplerian speed at which the dust orbits and v_ϕ is the orbital speed of the gas, such that it experiences a net centrifugal force that matches the combination of gravitational and radial pressure forces. In the case that we have a surface density enhance-

ment $\sim \Delta\Sigma$ over a radial length scale λ we can approximate the second term on the right hand side as $\sim c_s^2(R/\lambda)(\Delta\Sigma/\Sigma)$ (where c_s is the sound speed). Thus

$$v_\phi^2 = v_K^2 \left[1 + \left(\frac{c_s}{v_K} \right)^2 \frac{\Delta\Sigma R}{\Sigma \lambda} \right] \quad (3)$$

and since the second term on the right hand side is $\ll 1$, we may write

$$\Delta v = v_\phi - v_K \sim v_K \left(\frac{H}{R} \right)^2 \left(\frac{R}{\lambda} \right) \left(\frac{\Delta\Sigma}{\Sigma} \right) \quad (4)$$

where we have used the thin disc relation $H/R \sim c_s/v_K$. Therefore (since the maximum radial inflow rate is Δv) we find that the minimum time t_{\min} required to concentrate solid material in a pressure maximum is $\sim \lambda/\Delta v$ and thus:

$$t_{\min} = \lambda/\Delta v = \Omega^{-1} (\lambda/H)^2 (\Delta\Sigma/\Sigma)^{-1} \quad (5)$$

Since in a gravitationally unstable disc the scale length for density inhomogeneities is of order the most unstable wavelength, which, in marginally unstable discs is of order H (Toomre 1964, Lodato 2007), we can set $\lambda \sim H$ and hence

$$t_{\min} = \Omega^{-1} \left(\frac{\Delta\Sigma}{\Sigma} \right)^{-1} \quad (6)$$

2.2 The link between spiral arm amplitude and cooling timescale

Cossins et al. (2009) established, through analysis of a suite of simulations with various imposed ratios of cooling timescale to dynamical timescale, that the fractional amplitude of spiral features scales as the inverse square root of the cooling time:

$$\frac{\Delta\Sigma}{\Sigma} = \frac{1}{\sqrt{\Omega t_{\text{cool}}}} \quad (7)$$

This result can be simply understood, given that Cossins et al. (2009) also showed that the shocks in these discs are only marginally supersonic (i.e. with Mach number $= 1 + \epsilon$, where $\epsilon \ll 1$). The cooling timescales in gravitationally unstable discs are sufficiently long that cooling can be neglected *at the shock front* and thus we can invoke the result for the vertically averaged density contrast in weak adiabatic shocks:

$$\Delta\Sigma/\Sigma \propto \epsilon \quad (8)$$

Nevertheless cooling becomes important downstream of the shocks: in a state of thermal equilibrium we have the situation where the conversion of mechanical energy into heat that is achieved at the shock front is balanced by cooling downstream of the shock. The quantity of energy dissipated at the shock is proportional to the entropy jump at the shock which, in the case of weak adiabatic shocks, scales as ϵ^2 . We thus expect, in thermal equilibrium that

$$\epsilon \propto 1/\sqrt{t_{\text{cool}}} \quad (9)$$

and thus, combining equations (8) and (9)

$$\Delta\Sigma/\Sigma \propto 1/\sqrt{t_{\text{cool}}} \quad (10)$$

in agreement with the empirical result equation (7).

3 THE EFFICIENCY OF DUST AGGREGATION IN DISCS WITH REALISTIC COOLING

The results of Section 2 (equation (6)) imply that the minimum timescale for dust aggregation (normalised to the local orbital timescale) is of order the fractional density enhancement in the shock and this scales with the inverse square root of the ratio of the cooling timescale to dynamical timescale, as given by equation (7). We thus have

$$\Omega t_{\min} \simeq \sqrt{\Omega t_{\text{cool}}} \quad (11)$$

On the other hand, the lifetime of individual spiral features in self-gravitating discs is of order Ω^{-1} . We thus see that the extent to which dust can accumulate in a given spiral arm before this feature dissolves is related to the fractional density enhancement in the shock and hence on the local cooling timescale.

Figure 1 depicts contours of constant fractional density enhancement (and hence Ωt_{cool}) in the plane of radius versus steady state accretion rate for a disc around a solar mass object. These contours are constructed on the assumption that the transport properties of a marginally self-gravitating disc can be described as a pseudo-viscous process (see e.g. discussions in Cossins et al., 2009, Clarke 2009): specifically this allows one to relate the rate of energy dissipation to the local angular momentum transport (and hence radial mass flux) in the disc (see also Rafikov 2009). In a state of thermal equilibrium, this energy dissipated is balanced by cooling (on a timescale t_{cool}) so that (if one knows t_{cool} as a function of local conditions) one can relate t_{cool} to a local (pseudo-)viscosity. In Figure 1, we have simply replaced this local pseudo-viscosity by the corresponding accretion rate in the case that the disc was in a steady state: although this parameterisation depends on the steady state assumption, the local solutions (i.e. the relationship between local density, temperature and pseudo-viscosity in a given cooling regime) are not restricted to the steady state case. Our parameterisation however allows one to interpret solutions in terms of a quantity (i.e. accretion rate) that can be estimated in young stars.

The solutions on which Figure 1 is based assume that the disc cools radiatively: for the parameters of interest, the disc is optically thick and we hence model cooling via the radiative diffusion approximation, with the (Rosseland mean) opacity given by the piecewise power law fit as a function of density and temperature given by Bell & Lin (1994). Figure 1 shows the division of the (\dot{M}, R) plane according to the dominant opacity source (marked by dashed lines) and delineates the non-fragmenting marginally unstable regime by the two bold lines. To the right of this region, the ratio of cooling timescale to dynamical timescale is sufficiently short that the disc fragments (see e.g. Gammie 2001, Rice et al., 2005), rather than remaining in a self-regulated state of marginal gravitational stability: this line corresponds to the case where the fractional amplitude is about unity (Cossins et al., 2009). To the left, the dominant angular momentum transfer is provided by the magneto-rotational instability (MRI) and the disc is non-self gravitating. The dotted line indicates the contour where the fractional amplitude $\Delta\Sigma/\Sigma$ is 10 percent. For details concerning the construction of Figure 1, together with analytic expressions for the equilibrium solutions in various regimes and a discussion of our assumptions about the regions of the disc where the MRI is effective, see Clarke (2009).

We immediately see that the regions of the disc where large amplitude spiral features are to be expected are located at rather large radius (i.e. many tens of AU). Note the general argument given above that effective dust concentration implies that $\Delta\Sigma/\Sigma$ must not be *much* less than unity. The exact minimum value for

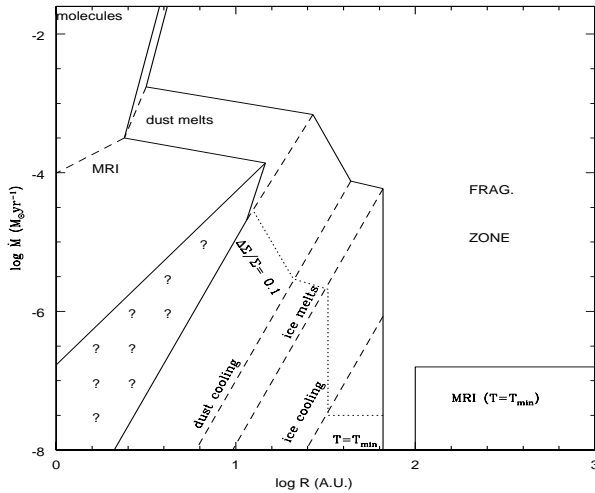


Figure 1. Regimes in the plane of steady state accretion rate versus radius in the case of a disc surrounding an object of mass $1M_{\odot}$, showing the dotted contour where the fractional surface density amplitude is 10%. The bold lines that parallel this dotted contour to the upper right represent the condition of unit fractional amplitude and also marks the boundary between fragmenting and non-fragmenting conditions in the disc gas. The region denoted with question marks is a region where the disc is expected to be too cold in order for the MRI to be active, but where a self-gravitating disc would be too hot for self-gravity to dominate angular momentum transport (full details can be found in Clarke, 2009).

$\Delta\Sigma/\Sigma$ required to induce substantial dust concentration cannot be obtained from the simple order of magnitude estimates provided above and needs to be obtained through detailed numerical simulations. Rice et al. (2004) obtain effective dust concentration in simulations where $\Omega t_{\text{cool}} = 7.5$, which corresponds to $\Delta\Sigma/\Sigma \approx 0.1$, thus placing an upper limit of around 10% to such minimum value. Unfortunately, it is not currently possible to perform self-consistent simulations of self-gravitating disc with Ωt_{cool} greater than around 10, because at this point the transport of angular momentum and dissipation of energy becomes dominated by numerical viscosity rather than self-gravity. Thus, in Figure 1 we show the line where the fractional amplitude is 10% as a guide to the region of the disc beyond which dust aggregation is likely to be effective. We note that the locus of constant fractional amplitude is independent of accretion rate for accretion rates over a range of values that are appropriate to young low mass stars (i.e. from $10^{-6}M_{\odot} \text{ yr}^{-1}$ to $< 10^{-7}M_{\odot} \text{ yr}^{-1}$): this is because the dominant opacity source is provided by ice grains, for which the opacity scales as T^2 and in this case the cooling timescale for a disc that is marginally gravitationally unstable turns out to be a function of radius only, independent of temperature (or accretion rate).

4 DISCUSSION

We have shown that the concentration of dust in spiral features is likely to be viable only in the outer regions of proto-planetary discs (i.e. beyond a few tens of AU), even though the disc may be self-gravitating at smaller radii. The inner extent of the zone of the disc where this mechanism is effective lies at a radius that is $\sim 3 - 10$ times smaller than the innermost radius where gas phase fragmentation of the disc is possible. We note that the recently imaged planets

in nearby young stars (Marois et al 2008, Kalas et al 2008) lie at orbital radii that would be consistent with the planetesimal creation mechanism discussed here.

We base our conclusion about viable regions of the disc on the fact that dust will only concentrate in spiral features if its concentration timescale is less than or of order the lifetime of individual features, which itself is of order the local orbital timescale. A short concentration timescale requires that the fractional amplitude of density enhancements in spiral arms is not much less than unity and this in turn implies that the local ratio of cooling timescale to dynamical timescale is sufficiently short. Figure 1 shows that the region of the disc that satisfies this requirement is at rather large radius and that the extent of this region is not a strong function of accretion rate in the disc. We emphasise that Rice et al. (2004) obtained successful dust concentration in spiral features because their toy cooling model was set up with an appropriately short cooling timescale.

As set out in the Introduction, the fact that solid material in the outer regions of self-gravitating discs may be efficiently converted into planetesimals may be critical for the retention of solids in the disc, since such planetesimals are immune to the migration effected by gas drag in the case of smaller grains. Pilot simulations of the dynamical behaviour of planetesimals in self-gravitating discs (Britsch et al., 2008) imply that the planetesimal orbits are driven to moderate eccentricities (of order 0.1) by interaction with fluctuating spiral features in the gas disc: the relatively large resultant velocity dispersion (of order a km/s), means that the runaway growth of this population in the direction of planet building is unlikely while the disc is still self-gravitating, but these velocities do suggest that collisions, when they occur, will be destructive and that small grains may be re-populated in the disc in this way (note that, since small dust is closely coupled to the gas, then the re-generated dust could then in principle re-enter the cycle of grain growth and - as long as the disc remains self-gravitating - accumulation in spiral shocks). Observationally, mm emission is detected in discs over many millions of years (i.e. long after the disc ceases to be self-gravitating), and shows no appreciable decline with time over this period (Andrews & Williams, 2005). Given the short radial migration timescales for such grains (less than a million years), this fact requires that they are replenished from some reservoir (see Takeuchi et al., 2005, Wyatt et al., 2007). The planetesimals assembled in the outer disc during the previous self-gravitating phase may provide a suitable stock pile of material from which small grains could be collisionally replenished. Before exploring such ideas in more detail (see e.g. Garaud, 2007) it is however first necessary to demonstrate that these planetesimals can be retained in the disc when subject to the dynamical evolution that results from interaction with the self-gravitating disc.

This issue of dynamical evolution also impacts on the other possible observational consequence of early planetesimal creation, i.e. the location of dust in debris disc systems. We have shown that we expect the creation of planetesimals in self-gravitating discs to be limited to large radii (i.e. a scale of tens of AU or more), which is a region similar in size to the planetesimal belts inferred in debris disc systems (Wyatt, 2008). (Note that, as mentioned in the Introduction, there are other candidate mechanisms for planetesimal assembly which may be operative in different regions of the disc: for example, some scenarios place the location of planetesimal assembly close to the ice-line, i.e. at a radius of ~ 3 A.U. in the case of solar type stars: see Kretke & Lin 2007, Brauer et al 2008). The mass in planetesimals that is deduced in the case of observed debris discs around sun-like stars is several tens of earth masses

and a similar figure is deduced from the requirement that Pluto and similar bodies can form on the same timescale as Neptune (Stern and Colwell 1997, Kenyon and Luu 1999), which necessarily implies formation in the early, gas rich phase of disc evolution.¹ Such masses represent a relatively small fraction of the total solid mass in the disc during the self-gravitating phase (which is of order hundreds of earth masses), implying that the planetesimal belts in debris disc systems may be a remnant of an initially larger population of planetesimals that were assembled in the self-gravitating phase. Again, further dynamical simulations are required in order to ascertain whether the ultimate distribution of planetesimals would retain a memory of their initial creation sites.

If such a memory were retained, then this would imply an inner hole in the planetesimal distribution at the time that the disc gas was dispersed, i.e. at the beginning of the debris disc phase. It is hard to establish whether or not debris discs initially contain planetesimal belts that extend in to small radii, since the short collisional lifetime at small radii means that the resultant dust should not be observable for long (Wyatt 2008). Current surveys (Carpenter et al 2009) have failed to detect hot dust (as evidenced by $8\mu\text{m}$ emission) in optically thin discs around solar type stars, but the relatively small number of young stars in the sample make it impossible to conclude from this that planetesimal belts at a scale of ~ 1 A.U. are necessarily absent in debris discs at birth. The statistics of debris discs with cool dust (associated with $24\mu\text{m}$ emission) also admits a variety of interpretations. According to the models of Kenyon and Bromley (2005), a disc with an inner hole in its planetesimal distribution should produce a *delayed* rise in (cool) dust production, since one has to wait for the formation of large planetesimals that can initiate a collisional cascade in smaller bodies. The formation timescale for large (~ 2000 km) planetesimals at 30 A.U. is relatively long (~ 10 Myr), so that according to this argument, debris should be absent in systems younger than ~ 10 Myr if primordial holes in the planetesimal distribution are common. However, the handful of $24\mu\text{m}$ detections in younger systems (Carpenter et al 2009) does not rule out this scenario, since these few objects could instead be the final remnants of primordial discs, where dust does not trace the location of a population of colliding planetesimals. Larger surveys studying the evolution of debris in younger stars are needed in order to constrain the architecture of planetesimal belts in solar type stars: it is notable that in the case of debris discs around A stars (where larger samples are available) there is some claimed evidence for a maximum brightness of debris at an age of 10 – 15 Myr (Currie et al 2008), which would be compatible with a ~ 30 A.U. hole in the planetesimal distribution in these objects.

In conclusion, we remark that it is often assumed that the spatial location of planetesimal belts is set by the orbital parameters of giant planets (see e.g. Quillen 2006 and the recent striking confirmation in the case of the Fomalhaut system of a suitably located planet in the debris disc: Kalas et al 2008). Here we merely point out that the location of planetesimal belts may also relate to the initial mechanism for planetesimal production. We have shown in this paper that the creation of planetesimals in self-gravitating discs is an outer disc phenomenon. The implication of this result for planet formation and the evolution of debris discs is yet to be explored.

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¹ A similar mass is also inferred in the case of the *primordial* Kuiper belt, from which it is then argued that the much smaller mass of the current belt is a consequence of depletion in a dynamical instability occurring up to a Gyr later: Gomes et al 2005